A Covariant Semi-Classical Theory of the Radiating Electron

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A new semi-classical equation of motion is suggested for the radiating electron. The characteristic length of the new theory is the Compton wavelength λ_c $(=\hbar/2\,m\,c)$ instead of the classical electron radius r_c $(=Z^2/2\,m\,c^2 \approx \frac{1}{137}\lambda_c)$, which is used in all purely classical theories of the radiating electron. However, the lowest order approximation of the radiation reaction contains only the classical radius r_c.

In a recent paper [1], we have investigated a non-local equation of motion for the radiating electron, and we have found quite agreeable properties of this new equation: there are neither runaway solutions nor pre-acceleration (i.e. acceleration before a force is switched-on), and a retarding friction force due to radiation reaction is always present even in one-dimensional, unlimited, constant force fields, where as well the Lorentz-Dirac equation as that of Mo and Papas do not exhibit the effect of radiation damping [1].

Nevertheless, two points are inherent in the new theory, which call for further improvement: the first one is the fact, that there exists still causality violation [2] in the sense, that the acceleration at a certain time is determined not only by the force at that same time but also by future forces. Though this causality violation is less severe than in the Lorentz-Dirac theory [3], it seems possible to remove this deficiency by a certain modification of the new equation of motion.

The second point, which we intend to improve hereafter refers to an argument, which can be applied against every classical theory of the radiating electron. In such a theory, the characteristic length dimension in the equation of motion is the classical electron radius r_c defined by

$$m_{\text{exp}}\,c^2 = Z^2/2\,r_c\,. \tag{1}$$

The range of non-local effects is determined by this length parameter. But if one thinks of quantum mechanics, the decisive length parameter in the description of the electron should be the Compton wavelength λ_c

$$\lambda_c = \frac{\hbar}{2\,m_{\text{exp}}\,c} = \frac{\hbar\,c}{Z^2}\cdot r_c\,, \tag{2}$$

which is roughly 137 times the classical radius r_c

$$Z^2/\hbar\,c = \alpha \approx 1/137 \tag{3}$$

(α ... Sommerfeld's fine structure constant).

This suggests that the non-locality parameter in the equation of motion in demand should have the order of magnitude of the Compton wavelength λ_c instead of the classical radius r_c.

These two heuristic considerations have led us to propose the following equation of motion for the semi-classical radiating electron

$$m_{\text{mech}}\,c^2\,\dot u^\lambda + m_{\text{el}}\,c^2\,[\hat{\dot u}^\lambda - (u\,\hat{\dot u})\,u^\lambda] = K^\lambda\,. \tag{4}$$

Here, m_{mech} designates that part of the mass of the electron, which cannot be identified with the mass equivalent of the classical Coulomb field energy. May be, m_{mech} is completely of non-electromagnetic nature or explainable only in the framework of quantum electrodynamics. The mass m_{el} is the mass equivalent of the classical Coulomb field surrounding the electron. The non-locality appears in the new Eq. (4) in the form of a shifted four-acceleration:

$$\hat{\dot u}^\lambda := \dot u^\lambda{}_{(s-\sigma)} \equiv \left.\frac{\mathrm{d}u^\lambda}{\mathrm{d}s}\right|_{s-\sigma}. \tag{5}$$

The mechanical part $m_{\text{mech}}\,c^2\,\dot u^\lambda$ does not contain the non-local effect

$$\dot u^\lambda \equiv \left.\frac{\mathrm{d}u^\lambda}{\mathrm{d}s}\right|_s$$

($\{u^\lambda\}$ ... four-velocity with $u^\lambda u_\lambda = +1$; s ... proper time). So we see that only the electromagnetic part exhibits a retarded response to the external force, but clearly a more detailed elaboration of this point is left for future work. Of course, we choose the non-locality parameter σ to be in the order of magnitude of the Compton wavelength λ_c: $\sigma \approx \lambda_c$. For the sake of preliminary definiteness, let us choose

$$\sigma = \tfrac{4}{3}\,\lambda_c\,. \tag{6}$$

Now we shall try to find the lowest order approximation of the new non-local Equation (4). In case of a trajectory, for which the unit tangent vector $\{u^\lambda\}$ does not change appreciably during proper time intervals of order σ, we can expand the shifted four-acceleration $\{\hat{\dot u}^\lambda\}$ as

$$\hat{\dot u}^\lambda = \dot u^\lambda{}_{(s-\sigma)} = \dot u^\lambda{}_{(s)} - \sigma\,\ddot u^\lambda{}_{(s)} + O_{(\sigma^2)}\,. \tag{7}$$

Hence, Eq. (4) becomes in this approximation

$$(m_{\text{mech}} + m_{\text{el}})\,c^2\,\dot u^\lambda - m_{\text{el}}\,c^2\,\sigma\,[\ddot u^\lambda + (\dot u\,\dot u)\,u^\lambda] = K^\lambda\,. \tag{8}$$

Since we put $\{K^\lambda\}$ to be the usual Lorentz force

$$K^\lambda = Z\,F^{\mu\lambda}\,u_\mu\,, \tag{9}$$

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Eq. (8) is just the Lorentz-Dirac equation of motion, if we require the constraint

$$m_{\text{el}}\, c^2\, \sigma = \tfrac{2}{3}\, Z^2 . \qquad (10)$$

Thus

$$m_{\text{el}}\, c^2 = \frac{Z^2}{2\, \lambda_{\text{c}}} = \alpha\, \frac{Z^2}{2\, r_{\text{c}}} \approx \frac{1}{137}\, m_{\text{exp}}\, c^2 , \qquad (11)$$

which means that the electromagnetic mass m_{el} is roughly 1/137 of the experimental mass m_{exp}, where

$$m_{\text{exp}} = m_{\text{mech}} + m_{\text{el}} . \qquad (12)$$

However, we know that the Lorentz-Dirac equation (8) with condition (10) does have solutions, for which the unit tangent vector $\{u^\lambda\}$ changes considerably over a proper time interval of length r_{c} (those are the runaway solutions and the pre-accelerative part of an otherwise reasonable solution). Therefore, in order to have a meaningful local approximation, we insert the neutral particle limit

$$m_{\text{exp}}\, c^2\, \dot{u}^\lambda = Z\, F^{\mu\lambda}\, u_\mu , \qquad (13)$$

into the Lorentz-Dirac equation (8) and obtain

$$m_{\text{exp}}\, c^2\, \dot{u}^\lambda - \tfrac{8}{3}\, r_c{}^2\, \{F^{\mu\lambda}\, F^{\nu}{}_{\mu}\, u_\nu + (F^{\mu\nu}\, F^{\sigma}{}_{\nu}\, u_\mu\, u_\sigma)\, u^\lambda\} = Z\, F^{\mu\lambda}\, u_\mu . \qquad (14)$$

Here, we have omitted a term of the form

$$\tfrac{4}{3}\, Z\, r_{\text{c}}\, \dot{F}^{\mu\lambda}\, u_\mu , \qquad (15)$$

because the variation of the external field over a proper time interval of length r_{c} must be very small otherwise we would not have the unit tangent vector $\{u^\lambda\}$ vary very little over time intervals of order $\sigma \approx 137\, r_{\text{c}}$.

For one-dimensional motion, however, the vector in brackets of Eq. (14) vanishes and therefore the term (15) should not be omitted in this case. With the usual substitution

$$\{u^\lambda\} = \{\text{Cosh}\, w;\, 0, 0, \text{Sinh}\, w\} \qquad (16)$$

one gets then ($F^{03} := E$)

$$m_{\text{exp}}\, c^2\, \dot{w}_{(s)} = Z\, E_{(s)} + \tfrac{4}{3}\, Z\, r_{\text{c}}\, \mathrm{d}E_{(s)}/\mathrm{d}s . \qquad (17)$$

This equation agrees with the non-covariant semiclassical result of Moniz and Sharp [4] [cf. their formula (12)]. Clearly, higher order approximations would not only involve the classical radius r_{c} but also the Compton wavelength λ_{c}.

Finally, let us write the semi-classical equation of motion (4) in one-dimensional form

$$m_{mech}\, c^2\, \dot{w}_{(s)} + m_{\text{el}}\, c^2\, \dot{w}_{(s-\sigma)}\, \text{Cosh}\, [w_{(s)} - w_{(s-\sigma)}] = K_{(s)} . \qquad (18)$$

From here it is seen easily that the invariant acceleration at a certain time s does not depend from the future values of the "velocity" $w_{(s)}$, rather from the past and simultaneous values of $w_{(s)}$, and therefore causality violation seems to be excluded in contrast to the former model [1]. Clearly, this point requires further analysis in the future.

The free equation (18) ($K_{(s)} \equiv 0$)

$$\dot{w}_{(s)} + (m_{\text{el}}/m_{\text{mech}})\, \dot{w}_{(s-\sigma)}\, \text{Cosh}\, [w_{(s)} - w_{(s-\sigma)}] = 0 \qquad (19)$$

suggests damped oscillations as solutions, because if $(m_{\text{el}}/m_{\text{ech}})\text{Cosh}[w_{(s)} - w_{(s-\sigma)}] < 1$ is valid for an initial interval, then the acceleration $\dot{w}$ at s is opposite in sign and smaller in amount with respect to $\dot{w}_{(s-\sigma)}$ for all times s following the initial interval. These oscillations can be interpreted as those of the coupled mechanical (m_{mech}) and electromagnetic (m_{el}) subsystems constituting the "electron" [5].

The new semi-classical equation of motion (4) seems to be a very promising one in various aspects and shall be studied further.

[1] M. Sorg, Z. Naturforsch. **31 a,** 1133 [1976].
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